The Sidereal System Revised in 1896. By Maxwell Hall.

The original article (*Memoirs R.A.S.* vol. xliii. p. 157) was revised in 1887 (*Monthly Notices*, vol. xlvii. p. 521); the present revision is undertaken to meet the determination by photography at the Potsdam Physical Observatory of the radial velocities of fifty-one stars, the determination by photography at the Oxford University Observatory of the annual parallaxes of about twenty-six stars of the second magnitude conveniently situated in that latitude, and to meet the recognition of the greater accuracy of the stellar proper motions as deduced by Dr. Auwers.

In the present revision the latter proper motions are always used when possible.

Note I. contains a list of those stars whose proper motions have been taken from other authorities.

Note II. contains a list of observed parallaxes; the means have been adopted.

Note III. contains the radial velocities observed by Dr. Huggins, at Greenwich, and at Potsdam. It is disappointing to find so many contradictory results; in the case of many stars the adopted means seem very uncertain.

It will be noticed that the general effect of the Potsdam observations is to greatly reduce all radial velocities; and it will be found that the Oxford parallaxes are much larger than the computed parallaxes; consequently it appears that there are large errors among the constants of the system hitherto adopted; and these large errors must first be ascertained before we can possibly proceed.

From the revision of 1887 the value of ΠV was found to be $0''\cdot10$; and from the manner in which ΠV and V enter the computations it will be seen that if ΠV be still considered $0''\cdot10$, but that if V be reduced to 5 and Π increased to $0''\cdot02$, then all the radial velocities will be diminished about one-half, and all the parallaxes will be nearly doubled.

Let us consider the effect of these changes on the very few stars whose parallaxes and radial velocities have both been ascertained with considerable accuracy:—

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Star.	Observed.			
	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$
	"		"	
Sirius	0·27	0 ?	0·30	0
Arcturus	0·07	− 9	0·38	− 7
Capella	0·07	+ 7	0·06	+ 4
Rigel	0·03	+ 4	0·03	+ 9
Vega	0·12	− 9	0·16	− 10
Procyon	0·23	+ 2	0·14	+ 4
α Orionis	0·02	+ 6	0·03	+ 7
Altair	0·20	− 8	0·32	− 7
Aldebaran	0·10	+ 11	0·04	+ 7
Polaris	0·05	− 6	0·02	− 6

Among these twenty results of computation the only serious difference between observation and computation occurs in the case of the parallax of *Arcturus*. Peters obtained 0″·127, Elkin 0″·016, so that the mean 0″·07 must yet be considered uncertain.

Omitting, then, the parallax of this star, the sums of the observed and computed parallaxes agree; and the sums of the observed and computed radial velocities, neglecting their signs, also agree.

In the following general computations there will be considerable saving of labour by employing only three decimal places for both natural numbers and logarithms instead of five, as before. In the case of any star the results depend upon its position and proper motion; and although the functions depending on position may be computed with accuracy, yet three places of decimals will clearly give as much numerical accuracy to the functions depending on proper motion as the uncertainty connected with proper motion will justify.

It was shown in the last revision that when failure occurred in computation such failure was generally due to the assumption of circularity of the star's orbit, and elliptic orbits were discussed.* We shall here give the cubic equation for u, so that in difficult cases it may be easily ascertained whether such failure exists or not—indeed, in cases where ρ_0 is not approximately known, much time may be saved by its use.

The cubic equation for u. $[v^2 = \mathrm{F}r_0^2.]$

We have $u = \dfrac{\mathrm{K} + a\Pi\mathrm{V}}{\mathrm{H} + \Pi\rho_0}$,

$$\text{and } \rho_0 = \frac{2\mathrm{V}\,(au - b + \mathrm{H}\Pi\mathrm{V})}{c^2 + u^2 - \Pi^2\mathrm{V}^2}.$$

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* In the fig. *Monthly Notices*, vol. xlvii. p. 522, *dele* the letter ψ; and *dele* the words "the angle ACS," p. 523, line 8.

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Multiply the last equation by Π, and add H to both sides, then

$$\mathrm{H}+\Pi\rho_0=\frac{2\Pi\mathrm{V}\,(au-b+\mathrm{H}\Pi\mathrm{V})+\mathrm{H}\,(c^2+u^2-\Pi^2\mathrm{V}^2)}{c^2+u^2-\Pi^2\mathrm{V}^2}.$$

Invert, and multiply by $(\mathrm{K}+a\Pi\mathrm{V})$; then

$$u=\frac{(\mathrm{K}+a\Pi\mathrm{V})\,(c^2+u^2-\Pi^2\mathrm{V}^2)}{2\Pi\mathrm{V}\,(au-b+\mathrm{H}\Pi\mathrm{V})+\mathrm{H}\,(c^2+u^2-\Pi^2\mathrm{V}^2)}.$$

Let

$$\mathrm{A}=\frac{\mathrm{K}-a\Pi\mathrm{V}}{\mathrm{H}};$$

$$\mathrm{B}=c^2+\Pi^2\mathrm{V}^2-\frac{2b\,\Pi\mathrm{V}}{\mathrm{H}};$$

$$\mathrm{C}=\frac{\mathrm{K}+a\Pi\mathrm{V}}{\mathrm{H}}\,(c^2-\Pi^2\mathrm{V}^2);$$

then

$$u^3-\mathrm{A}u^2+\mathrm{B}u-\mathrm{C}=0.$$

This equation may easily be solved by approximation, and then

$$\rho_0=\frac{\mathrm{K}+a\Pi\mathrm{V}-\mathrm{H}u}{\Pi u}.$$

Up to the present we have assumed that the stars are uniformly distributed, so that the force to the centre varies directly as r_0, and so that in circular orbits

$$v^2=\mathrm{F}r_0^{\,2}.$$

But from considerations based upon the number and magnitude of the stars there is good reason to suppose that the number of the stars in an unit volume of space, or, in other words, that the star-density of space, varies inversely as the distance from the centre.

In this case the force to the centre is constant, and for circular orbits

$$v^2=\mathrm{F}r_0.$$

Now this change in the law of force has little or no effect on the nearer stars; and in order to ascertain which law of force gives the better results we shall have to compute the parallaxes and radial velocities of the stars upon the two suppositions.

It will be convenient to collect all the formulæ for computation:

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$$\left.\begin{array}{l}\theta = \text{North Polar Distance}\\ \phi = \text{Right Ascension}\end{array}\right\}\text{of a star, Jan. 1·0, 1850;}$$

$$\left.\begin{array}{l}\delta\theta = \text{Annual proper motion in } \theta\\ \delta\phi = \text{Annual proper motion in } \phi\end{array}\right\}\text{in seconds of arc;}$$

ϖ = Annual parallax, in seconds of arc;

ρ_0 = Distance in sidereal units $= \frac{1}{\varpi}$;

$\frac{d\rho}{dt}$ = Annual change in distance, taking the radius of the Earth's orbit as the unit of space.

$$l = \sin\theta\cos\phi\,; \quad \delta l = n\delta\theta\,.\,\cos\phi - m\delta\phi\,;$$
$$m = \sin\theta\sin\phi\,; \quad \delta m = n\delta\theta\,.\,\sin\phi + l\delta\phi\,;$$
$$n = \cos\theta\,; \quad \delta n = -\sin\theta\,.\,\delta\theta.$$

$$a = l\lambda + m\mu + n\nu\,; \quad \log\lambda = -9{\cdot}166$$
$$b = \lambda\delta l + \mu\delta m + \nu\delta n\,; \quad \log\mu = -9{\cdot}914$$
$$c^2 = \delta l^2 + \delta m^2 + \delta n^2\,; \quad \log\nu = +9{\cdot}744.$$

$$H = lL + mM + nN\,; \quad \log L = -9{\cdot}936$$
$$K = L\delta l + M\delta m + N\delta n\,; \quad \log M = -9{\cdot}223$$
$$\log N = -9{\cdot}678.^{*}$$

$$V = 5\,; \quad \Pi V = 0{\cdot}100\,; \quad \Pi = 0{\cdot}020.$$

$$u = \frac{K + a\Pi V}{H + \Pi\rho_0}\,; \quad \frac{d\rho}{dt} = -u\rho_0.$$

Then for $v^2 = Fr_0^2$ we have

$$\rho_0 = \frac{2V(au - b + H\Pi V}{c^2 + u^2 - \Pi^2 V^2}.$$

And for $v^2 = Fr_0$ we have

$$\rho_0 = \frac{2V\left(au - b + \dfrac{H\Pi V}{1 + \Pi r_0}\right)}{c^2 + u^2 - \dfrac{\Pi^2 V^2}{1 + \Pi r_0}}$$

$$\text{where } r_0^2 = R_0^2 + \rho_0^2 + 2HR_0\rho_0.$$

$$\text{or } \Pi r_0 = \sqrt{1 + \Pi^2\rho_0^2 + 2H\Pi\rho_0}.$$

The following example for $v^2 = Fr_0^2$ is given. Much labour may be saved by using printed forms.

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[*] Instead of $-9{\cdot}677$, so that $L^2 + M^2 + N^2$, to three decimal places, may equal 1·000.

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[EXAMPLE.]

a Lyræ. M = 0·2

R.A. = $18^h\ 31^m\ 51^s$; P.M. in R.A. = $+0^s{\cdot}0173$ } Auwers
N.P.D. = $51°\ 21'$; P.M. in N.P.D. = $-0''{\cdot}295$ }

$\theta = 51°\ 21'$; $\delta\theta = -0{\cdot}295$
$\phi = 277\ 58$; $\delta\phi = +0{\cdot}260$

log sin $\theta = +9{\cdot}893$ $+9{\cdot}893$
log cos $\phi = +9{\cdot}142$; log sin $\phi = -9{\cdot}996$
log $l = +9{\cdot}035$; log $m = -9{\cdot}889$; log $n = +9{\cdot}796$

log $n = +9{\cdot}796$
log $\delta\theta = -9{\cdot}470$
$-9{\cdot}266$
log cos $\phi = +9{\cdot}142$
$-8{\cdot}408 = -{\cdot}026$

$-9{\cdot}266$
log sin $\phi = -9{\cdot}996$
$+9{\cdot}262 = +{\cdot}183$

log $m = -9{\cdot}889$
log $\delta\phi = +9{\cdot}415$
$-9{\cdot}304^* = +{\cdot}201$
$\delta l = +{\cdot}175$

log $l = +9{\cdot}035$
$+9{\cdot}415$
$+8{\cdot}450 = +{\cdot}028$
$\delta m = +{\cdot}211$

log $\delta l = +9{\cdot}243$; $\delta l^2 = {\cdot}030\ 6$
log $\delta m = +9{\cdot}324$; $\delta m^2 = {\cdot}044\ 5$
log $\delta n = +9{\cdot}363$; $\delta n^2 = {\cdot}053\ 2$
$c^2 = {\cdot}128$

log $\lambda = -9{\cdot}166$
log $l = +9{\cdot}035$
$-8{\cdot}201 = -{\cdot}016$

log $\lambda = -9{\cdot}166$
log $\delta l = +9{\cdot}243$
$-8{\cdot}409 = -{\cdot}026$

log $\mu = -9{\cdot}914$
log $m = -9{\cdot}889$
$+9{\cdot}803 = +{\cdot}635$

log $\mu = -9{\cdot}914$
log $\delta m = +9{\cdot}324$
$-9{\cdot}238 = -{\cdot}173$

log $\nu = +9{\cdot}744$
log $n = +9{\cdot}796$
$+9{\cdot}540 = +{\cdot}347$
$a = +{\cdot}966$

log $\nu = +9{\cdot}744$
log $\delta n = +9{\cdot}363$
$+9{\cdot}107 = +{\cdot}128$
$b = -{\cdot}071$

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* Change the sign.

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$$
\begin{aligned}
\log L &= -9{\cdot}936\\
\log l &= +9{\cdot}035\\
\hline
-8{\cdot}971 &= -{\cdot}094
\end{aligned}
\qquad
\begin{aligned}
\log L &= -9{\cdot}936\\
\log \delta l &= +9{\cdot}243\\
\hline
-9{\cdot}179 &= -{\cdot}151
\end{aligned}
$$

$$
\begin{aligned}
\log M &= -9{\cdot}223\\
\log m &= -9{\cdot}889\\
\hline
+9{\cdot}112 &= +{\cdot}130
\end{aligned}
\qquad
\begin{aligned}
\log M &= -9{\cdot}223\\
\log \delta m &= +9{\cdot}324\\
\hline
-8\ 547 &= -{\cdot}035
\end{aligned}
$$

$$
\begin{aligned}
\log N &= -9{\cdot}678\\
\log n &= +9{\cdot}796\\
\hline
-9{\cdot}474 &= -{\cdot}298\\
H &= -{\cdot}262
\end{aligned}
\qquad
\begin{aligned}
\log N &= -9{\cdot}678\\
\log \delta n &= +9{\cdot}363\\
\hline
-9{\cdot}041 &= -{\cdot}110\\
K &= -{\cdot}296\\
a\Pi V &= +{\cdot}097\\
\hline
\left\{\begin{aligned} K + a\Pi V &= -\ {\cdot}199\\ \log. &= -9{\cdot}299\end{aligned}\right.
\end{aligned}
$$

$$
\begin{aligned}
\text{Assume } \rho_0 &= \ 6\ 4\\
\Pi\rho_0 &= \ {\cdot}128\\
H + \Pi\rho_0 &= -\ {\cdot}134\,;
\end{aligned}
\qquad
\begin{aligned}
\log &= -\ 9{\cdot}127\\
\hline
\log u &= +10{\cdot}172\\
\log a &= +\ 9{\cdot}985\\
\hline
\log au &= +10{\cdot}157
\end{aligned}
$$

$$
\begin{aligned}
c^2 &= 0{\cdot}128\\
u^2 &= 2{\cdot}208\\
-\Pi^2V^2 &= -{\cdot}010\\
\hline
&\ 2{\cdot}326
\end{aligned}
\qquad
\begin{aligned}
au &= +1{\cdot}436\\
-b &= +\ {\cdot}071\\
H\Pi V &= -\ {\cdot}026\\
\hline
&\ 1{\cdot}481
\end{aligned}
$$

And $$\frac{1{\cdot}481 \times 10}{2{\cdot}326} = 6{\cdot}4 = \rho_0$$

$$
\left\{\begin{aligned}
&\text{Therefore } \varpi = 0''{\cdot}16\\
&\text{and } \frac{d\rho}{dt} = -10
\end{aligned}\right.
$$

No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed.				Notes.
									$v^2=\mathrm{F}r_0^2$		$v^2=\mathrm{F}r_0$		
							ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	
			h m s	s	° ′	″	″		″		″		
1	α Canis Maj. (*Sirius*)	−1·2	6 38 32	−0·0372	106 31	+1·199	0·27	0 ?	0·30	0	0·30	0	$\frac{d\rho}{dt}$ var.
2	α Argûs (*Canopus*)	−0·4	6 20 37	·0000	142 37	0·000	0·03	...	...	...	...	...	The P.M.'s are zero.
3	α Boötis (*Arcturus*)	0·0	14 8 49	− ·0799	70 2	+1·977	0·07	− 9	0·38	− 7	0·38	− 7	
4	α Centauri	0·0	14 29 27	− ·4842	150 13	−0·747	0·72	...	large	− 1	large	− 1	$\rho_0=0·6$, $\varpi=1''·67$, in both cases.
5	α Aurigæ (*Capella*)	0·1	5 5 37	+ ·0079	44 10	+0·424	0·07	+ 7	0·06	+ 4	0·06	+ 5	
6	β Orionis (*Rigel*)	0·2	5 7 20	− ·0012	98 23	−0·005	0·03	+ 4	0·03	+ 9	0·03	+ 9	
7	α Lyræ (*Vega*)	0·2	18 31 51	+ ·0173	51 21	−0·295	0·12	− 9	0·16	−10	0·16	−10	
8	α Canis Min. (*Procyon*)	0·5	7 31 27	− ·0474	84 24	+1·027	0·23	+ 2	0·14	+ 4	0·14	+ 4	
9	α Orionis	0·9	5 47 3	+ ·0008	82 38	−0·024	0·02	+ 6	0·03	+ 7	0·03	+ 7	
10	α Aquilæ (*Altair*)	1·0	19 43 28	+ ·0351	81 32	−0·384	0·20	− 8	0·32	− 7	0·32	− 7	$\frac{d\rho}{dt}$ var.
11	α Tauri (*Aldebaran*)	1·1	4 27 19	+ ·0035	73 48	+0·184	0·10	+11	0·04	+ 7	0·04	+ 7	
12	α Virginis (*Spica*)	1·1	13 17 18	− ·0044	100 23	+0·018	...	− 4	...	...	0·00	− 2	$\rho_0=265$; $r_0=298$.
13	β Gemin. (*Pollux*)	1·2	7 36 8	− ·0481	61 37	+0·051	0·06	− 7 ?	...	...	...	...	
14	α Leonis (*Regulus*)	1·3	10 0 23	− ·0182	77 18	−0·018	0·09	+ 1	0·03	+ 5	0·04	+ 4	
15	α Cygni	1·4	20 36 19	− ·0003	45 15	−0·003	0·01	− 7	0·01	− 8	0·01	− 8	
16	α Gemin. (*Castor*)	1·5	7 25 1	− ·0151	57 47	+0·079	...	+ 1 ?	...	...	...	...	

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No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed.				Notes.
									$v^2=Fr_0^2$		$v^2=Fr_0$		
							ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	
			h m s	s	° ′	″	″		″		″		
17	γ Orionis	1·8	5 17 5	+ ·002	83 47	+0·04	...	+ 2	0·04	+ 8	0·04	+ 8	
18	ε Orionis	1·8	5 28 36	− ·0018	91 18	−0·006	...	+ 6	0·03	+ 9	0·03	+ 9	
19	ζ Orionis	1·8	5 33 12	− ·0008	92 2	−0·010	...	+ 3	0·03	+ 9	0·03	+ 9	
20	ε Ursæ Maj.	1·8	12 47 25	+ ·0115	33 14	+0·021	0·08	− 1 ?	0·06	− 3	0·07	− 3	
21	α Persei	1·9	3 13 38	+ ·0015	40 41	+0·033	0·09	− 6	0·02	+ 1	0·02	+ 2	K zero; computed $\frac{d\rho}{dt}$ doubtful.
22	β Tauri	1·9	5 16 49	+ ·0013	61 31	+0·180	0·06	− 2 ?	0·04	+ 7	0·04	+ 7	
23	α Ursæ Maj.	1·9	10 54 26	− ·0180	27 26	+0·071	0·05	− 9	0·16	− 6	0·15	− 6	
24	η Ursæ Maj.	1·9	13 41 37	− ·0115	39 56	+0·014	...	− 6	0·01	− 5	0·01	− 5	
25	β Aurigæ	2·0	5 48 32	− ·0065	45 4	+0·011	0·06	− 6	0·01	+ 5	...	...	
26	α Androm.	2·1	0 0 39	+ ·0095	61 44	+0·156	0·06	− 4 ?	0·03	− 1	0·03	− 1	
27	α Ursæ Min. (*Polaris*)	2·1	1 5 1	+ ·1109	1 29	+0·002	0·05	− 6	0·02	− 6	0·02	− 6	
28	γ Androm.	2·1	1 54 43	+ ·0021	48 24	+0·051	...	− 3	0·02	+ 1	0·00	0	For Fr_0, $\rho_0=300$.
29	α Arietis	2·1	1 58 44	+ ·0127	67 15	+0·134	0·08	− 4	0·03	+ 3	0·03	+ 4	
30	γ Gemin.	2·1	6 29 3	+ ·0023	73 29	+0·035	...	− 4	0·10	+ 6	0·10	+ 6	
31	γ Cassiopeæ	2·2	0 47 42	+ ·0013	30 6	+0·015	0·01	− 4	0·02	− 3	0·01	− 5	

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No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed. $v^2 = Fr_0^2$		Computed. $v^2 = Fr_0$		Notes.
			h m s	s	° ′	″	ϖ ″	$\frac{d\rho}{dt}$	ϖ ″	$\frac{d\rho}{dt}$	ϖ ″	$\frac{d\rho}{dt}$	
32	β Androm.	2·2	1 1 21	+ ·0144	55 11	+0·084	0·07	+ 2	0·03	+ 1	0·03	+ 1	
33	δ Orionis	2·2	5 24 21	− ·0014	90 25	+0·005	...	0	0·03	+ 9	0·03	+ 9	
34	γ Leonis	2·2	10 11 42	+ ·0208	69 24	+0·136	...	− 9	...	...	...	...	
35	β Leonis	2·2	11 41 24	− ·0356	74 35	+0·098	0·03	− 3	0·05	+ 1	0·05	+ 1	
36	ζ Ursæ Maj.	2·2	13 17 53	+ ·0134	34 17	+0·022	...	0?	0·07	− 3	0·08	− 3	
37	β Ursæ Min.	2·2	14 51 12	− ·0077	15 14	+0·005	0·02	+ 3	0·03	− 8	0·03	− 8	$\frac{d\rho}{dt}$ var.
38	α Ophiuchi	2·2	17 27 58	+ ·0066	77 20	+0·217	...	0?	0·03	− 5	0·03	− 5	
39	α Coronæ	2·3	15 28 20	+ ·0085	62 47	+0·094	...	+ 7	0·02	− 3	0·04	− 3	
40	γ Cygni	2·3	20 16 51	− ·0001	50 13	−0·020	0·10	− 3	0·02	− 8	0·02	− 8	
41	β Cassiopeæ	2·4	0 1 12	+ ·0658	31 41	+0·190	0·16	0	0·06	− 4	0·06	− 4	
42	α Cassiopeæ	2·4	0 32 2	+ ·0035	34 17	+0·038	0·04	− 3	0·02	− 3	0·02	− 4	
43	β Persei (*Algol*)	2·4	2 58 26	− ·0017	49 38	−0·010	0·05	0	0·01	+ 2	0·01	+ 2	
44	β Ursæ Maj.	2·4	10 52 45	+ ·0085	32 49	−0·047	0·08	+ 1?	0·21	− 1	0·23	− 1	
45	γ Ursæ Maj.	2·4	11 45 55	+ ·0098	35 28	−0·008	0·10	0?	0·09	− 2	0·11	− 2	
46	γ Draconis	2·4	17 53 8	− ·0018	38 29	+0·028	0·07	− 2	0·03	− 9	0·02	− 9	
47	ε Pegasi	2·4	21 36 49	+ ·0008	80 49	−0·011	0·08	− 2?	0·02	− 4	...	...	
48	α Pegasi	2·4	22 57 18	+ ·0028	75 36	+0·030	0·08	− 4?	0·02	− 3	...	...	

No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed. ϖ	Observed. $\frac{d\rho}{dt}$	Computed. $v^2 = Fr_0^2$ ϖ	Computed. $v^2 = Fr_0^2$ $\frac{d\rho}{dt}$	Computed. $v^2 = Fr_0$ ϖ	Computed. $v^2 = Fr_0$ $\frac{d\rho}{dt}$	Notes.
			h m s	s	° ′	″	″		″		″		
49	ε Boötis	2·6	14 38 26	− ·0043	62 17	−0·001	...	− 4	...	...	...	...	
50	α Cephei	2·6	21 15 0	+ ·0211	28 3	−0·025	0·06	−14	0·04	− 8	0·03	− 8	
51	β Pegasi	2·6	22 56 30	+ ·0130	62 44	−0·133	...	+ 1	0·45	0	0·08	− 1	
52	β Libræ	2·7	15 8 56	− ·0079	98 50	+0·017	...	− 2	0·00	− 6	0·01	− 4	For Fr_0^2, $\rho_0 = 375$.
53	α Serpentis	2·7	15 36 53	+ ·0079	83 6	−0·056	...	+ 5	...	...	...	...	
54	ε Cygni	2·7	20 40 9	+ ·0280	56 35	−0·335	0·13	...	0·10	− 6	0·10	− 6	
55	δ Leonis	2·8	11 6 7	+ ·0102	68 39	+0·115	...	− 3	0·21	− 1	0·80	0	
56	β Herculis	2·8	16 23 46	− ·0090	68 11	+0·015	...	− 8	0·01	− 9	0·02	− 8	
57	ι Ursæ Maj.	3·2	8 48 55	− ·0441	41 22	+0·247	0·13	...	large	− 2	large	− 2	$\rho_0 = 0·6$, $\varpi = 1''·67$, in both cases.
58	θ Ursæ Maj.	3·2	9 22 48	− ·1041	37 39	+0·564	0·05	...	large	− 3	large	− 3	$\rho_0 = 0·4$, $\varpi = 2''·63$, in both cases.
59	α Herculis	3·2	17 7 49	− ·0019	75 26	−0·030	0·05	...	...	...	...	...	
60	δ Herculis	3·3	17 8 52	− ·005	64 59	+0·15	0·08	...	0·04	−10	0·04	−10	
61	π Herculis	3·4	17 9 50	− ·0035	53 1	−0·005	0·11	...	large	−10	large	−10	$\rho_0 = 1·0$, $\varpi = 1''·00$, in both cases.
62	η Cassiopeæ	3·5	0 40 3	+ ·1346	32 59	+0·481	0·31	...	0·14	− 2	0·13	− 2	
6	η Herculis	3·7	16 37 45	+ ·0028	50 47	+0·077	0·40	...	...	...	...	...	

No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed. $v^2 = Fr_0^2$		Computed. $v^2 = Fr_0$		Notes.
							ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	
			h m s	s	° ′	″	″		″		″		
64	10 Ursæ Maj.	4·2	8 50 53	− ·036	47 38	+0·28	0·20	...	large	+ 1	large	+ 1	ρ_0 = 0·21, ϖ = 4″·76, in both cases. This star is B A C 3059.
65	70 (*p*) Ophiuchi	4·3	17 57 53	+ ·0131	87 28	+1·109	0·22	...	0·22	− 9	0·22	− 9	
66	θ Cassiopeæ	4·4	1 2 0	+ ·0233	35 39	+0·024	0·23	...	0·05	+ 1	0·04	+ 2	
67	e Eridani	4·4	3 13 55	+ ·270	133 39	−0·75	0·14	...	0·67	+ 9	0·67	+ 9	
68	o² Eridani	4·4	4 8 22	− ·1442	97 53	+3·442	0·19	...	large	+ 1	large	+ 1	ρ_0 = 0·90, ϖ = 1″·10, and ρ_0 = 0·93, ϖ = 1″·08, respectively
69	δ Ursæ Min.	4·4	18 20 44	+ ·0256	3 24	−0·040	0·03	...	0·03	− 6	0.03	− 6	
70	δ Equulei	4·6	21 7 11	+ ·012	80 36	+0·30	0·04	...	0·09	+ 1	0·08	+ 1	
71	σ Draconis	4·7	19 32 38	+ ·0973	20 36	+1·766	0·25	...	large	0	large	0	ρ_0 = 0·23, ϖ = 4″·35, and ρ_0 = 0·28, ϖ 3″·57, respectively
72	ν² Draconis	4·8	17 29 19	+ ·0179	34 43	−0·044	0·28	...	0·11	−10	0·11	−10	
73	ν¹ Draconis	4·9	17 29 14	+ ·0183	34 43	−0·048	0·32	...	0·12	− 9	0·12	− 9	
74	ζ Toucani	5·0	0 12 12	+ ·273	155 45	−1·16	0·06	...	large	+ 7	large	+ 7	ρ_0 = 0·39, ϖ = 2″.54, in both cases.
75	61 Cygni	5·0	21 0 11	+ ·3444	51 59	−3·230	0·43	...	0·88	− 5	0·87	− 5	
76	μ Cassiopeæ	5·2	0 58 19	+ ·3860	35 49	+1·580	0·18	...	0·40	− 1	0·40	− 1	

No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed.				Notes.
									$v^2=Fr_\circ^2$		$v^2=Fr_\circ$		
							ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	
			h m s	s	° ′	″	″		″		″		
77	ε Indi	5·2	21 51 51	+ ·480	147 24	+ 2·60	0·22	...	0·58	+ 5	0·58	+ 5	
78	Cephei 51 (Hev.)	5·3	6 28 33	− ·0400	2 45	+ 0·051	0·03	...	0·03	− 6	0·03	− 6	
79	3077 Bradley	5·5	23 6 4	+ ·249	33 40	− 0·287	0·18	...	0·25	− 3	0·25	− 3	
80	85 Pegasi	5·8	23 54 21	+ ·0622	63 43	+ 0·980	0·06	...	0·14	− 1	0·14	− 1	
81	20 Leonis Min.	6·0	9 52 21	− ·0431	57 20	+ 0·427	0·06	...	0·12	− 8	0·12	− 8	
82	1830 Groombridge	6·4	11 44 19	+ ·3461	51 12	+ 5·776	0·13	...	large	− 3	large	− 3	$\rho_0=0·18$, $\varpi=5''·56$, in both cases.
83	1618 Groombridge	6·5	10 2 6	− ·1390	39 48	+ 0·501	0·25	...	large	0	large	0	$\rho_0=0·006$ and 0·005.
84	Σ 1516	7·0	11 5 16	− ·1029	15 43	− 0·117	0·15	...	0·21	− 7	0·21	− 7	
85	21185 Lalande	7·5	10 55 8	− ·0453	53 2	+ 4·716	0·46	...	large	− 4	large	− 4	$\rho_0=0·4$, $\varpi=2''·50$, in both cases.
86	9352 Lacaille	7·5	22 56 0	+ ·5672	126 43	− 1·306	0·28	...	large	− 1	large	− 1	$\rho_0=0·115$ in both cases
87	Σ 2398	7·7	18 41 0	− ·1726	30 35	− 1·896	0·35	...	...	...	...	...	
88	34 Groombridge	8·1	0 9 52	+ ·2551	46 50	− 0·366	0·31	...	0·39	− 1	0·38	− 1	
89	21 258 Lalande	8·5	10 58 0	− ·4004	45 42	− 0·943	0·23	...	...	...	...	...	
90	17 415 Oelt. Argel.	9·0	17 37 17	− ·0679	21 33	+ 1·213	0·20	...	...	...	...	...	
91	11 677 Oelt. Argel.	9·5	11 12 0	− 0·507	23 20	− 0·21	0·18	...	large	− 7	large	− 7	$\rho_0=0·6$, $\varpi=1''·67$, in both cases.

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It thus appears that the results from $v^2 = \mathrm{F}r_0^2$ are identical with those from $v^2 = \mathrm{F}r_0$ in 50 cases; that the results from the former equation are better than those from the latter in 10 cases; but that the differences are important in only 4 cases;[*] that the results from the latter equation are better than those from the former in 11 cases, but that the difference is important in only 1 case;[†] and that in the remaining 20 cases the results are either unknown or doubtful.

The present researches therefore show that the results from the two equations are nearly the same, but that those from

$$v^2 = \mathrm{F}r_0^2$$

are slightly preferable to those from the other; and in consequence $v^2 = \mathrm{F}r_0^2$ should certainly be adopted, although no doubt in any special case, as in No. 12, a solution may be obtained from $v^2 = \mathrm{F}r_0$, the circumstance being duly noted.

The results of the above computations may be thus roughly analysed:—

Of the 72 stars observed for parallax, in 49 cases the computations are more or less correct, in 15 cases doubtful, in 1 case wrong, and in 7 cases impossible.

Of the 53 stars observed for radial velocity, in 28 cases the computations are more or less correct, in 13 cases doubtful, in 5 cases wrong, and in 7 cases impossible.

This is a very great improvement upon the revision of 1887, and we are therefore encouraged to look for still closer agreement between observation and theory; at present 3 stars out of every 5 revolve in orbits more or less circular round the common centre, and 2 stars out of every five revolve in orbits more or less elliptic; and it is of course impossible to say at present what degree of circularity will be ultimately obtained.

In our search for further improvement the first matter for consideration is the position of the solar apex, or the point towards which the Sun moves; from the earlier investigations we found

$$\left.\begin{array}{l} \alpha = 259^{\circ}\!\cdot\!8 \\ \beta = 56\cdot4 \end{array}\right\} 1850;$$

but recent investigations show that while β is tolerably correct, α should be considerably increased:—

	α	β		α	β
Dunkin	263°·7	65°·0	L. Struve	273°·3	62°·7
L. de Ball	269·0	66·8	Plummer	273·1	66·6
Rancken	284·6	58·1	Boss	283·3	45·9
Bischof	285·2	41·5	Stumpe	285·0	53·8
Ubaghs	262·4	63·4	Porter	281·2	49·3

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[*] Nos. 25, 47, 48, and 55.

[†] No. 12.

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But if we still assume that the solar orbit is circular, a considerable increase in a will require a considerable increase in the R.A. of the centre; and this is out of the question in the present state of the theory.

It will therefore be advisable to find corrections to the adopted positions of the Centre and Apex, removing the restriction that they must be 90° apart; but as this will involve 5 corrections, it will be further advisable to attempt the complete solution and find the 7 corrections, $\Delta\lambda$, $\Delta\mu$, ΔL, ΔM, ΔV, $\Delta\Pi$, and $\Delta\Lambda$, the necessity of which was foreseen in the Revision of 1887, page 539.

We now have

$$F = \Pi^2 V^2 (1 - \Lambda);$$

$$v^2 = F r_0^2;$$

$$r_0^2 = R_0^2 + 2HR_0\rho_0 + \rho_0^2;$$

$$u = \frac{K + a\Pi V + \varpi\Gamma V}{H + \Pi\rho_0};$$

$$\Gamma = L\lambda + M\mu + N\nu;$$

$$\rho_0 = \frac{2V\ (au - b + H\Pi V)}{c^2 + u^2 - \Pi^2V^2 + \Lambda\varpi^2V^2 \,.\, \Pi^2 r_0^2};$$

$$\frac{d\rho}{dt} + u\rho_0 = 0;$$

$$\varpi = \frac{1}{\rho_0}.$$

With regard to the factor $\Pi^2 r_0^2$ in the term whose coefficient is Λ, whenever ϖ is equal to or greater than 0″·1, $\Pi^2 r_0^2$ may be taken equal to 1.

Let

$$\epsilon = 2\varpi V\ (au - b + H\Pi V) - (c^2 + u^2 - \Pi^2V^2) - \Lambda \,.\, \varpi^2V^2 \,.\, \Pi^2 r_0^2,$$

so that if all the quantities have been rightly taken

$$\epsilon = 0.$$

Again let

$$S = \frac{\varpi\left(aV + \frac{d\rho}{dt}\right)}{H + \Pi\rho_0}$$

$$P = \frac{L}{N}; \quad Q = \frac{M}{N};$$

$$p = \frac{\lambda}{\nu}; \quad q = \frac{\mu}{\nu};$$

$$U = \delta l - P\delta n - u\ (l - nP) + \varpi V\ (\lambda - \nu P);$$

$$W = \delta m - Q\delta n - u\ (m - nQ) + \varpi V\ (\mu - \nu Q);$$

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then

$$\frac{d\epsilon}{d\varpi} = 2V(au - b + H\Pi V) - 2S\left(\frac{d\rho}{dt} \,.\, \Pi\rho_0 - \Gamma V) - 2\Lambda V^2(\varpi + H\Pi\right);$$

$$\frac{d\epsilon}{d\lambda} = 2\varpi V\{u(l - np) - (\delta l - p\delta n)\} + 2SV\{\Pi(l - np) + \varpi(L - Np)\};$$

$$\frac{d\epsilon}{d\mu} = 2\varpi V\{u(m - nq) - (\delta m - q\delta n)\} + 2SV\{\Pi(m - nq) + \varpi(M - Nq)\};$$

$$\frac{d\epsilon}{dL} = 2SU + 2\varpi V \,.\, \Pi V(l - nP)(1 - \Lambda);$$

$$\frac{d\epsilon}{dM} = 2SW + 2\varpi V \,.\, \Pi V(m - nQ)(1 - \Lambda);$$

$$\frac{d\epsilon}{dV} = 2\varpi(au - b + H\Pi V) + 2S(\Pi a + \varpi\Gamma) + 2\Pi V(\Pi + H\varpi) - 2\Lambda\varpi^2 V \,.\, \Pi^2 r_0^2;$$

$$\frac{d\epsilon}{d\Lambda} = -\varpi^2 V^2 \,.\, \Pi^2 r_0^2;$$

$$\frac{d\epsilon}{d\Pi} = 2S\left(aV + \frac{d\rho}{dt}\right) + 2\varpi V^2(H + \Pi\rho_0)(1 - \Lambda);$$

where both Γ and Λ are zero at present.

Again let

$$\frac{d\epsilon}{d\lambda} = \mathfrak{A}\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{d\mu} = \mathfrak{B}\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{dL} = A\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{dM} = B\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{dV} = C\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{d\Lambda} = D\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{d\Pi} = E\frac{d\epsilon}{d\varpi};$$

then

$$\mathfrak{A} \,.\, \Delta\lambda + \mathfrak{B} \,.\, \Delta\mu + A \,.\, \Delta L + B \,.\, \Delta M + C \,.\, \Delta V + D \,.\, \Delta\Lambda + E \,.\, \Delta\Pi + \Delta\varpi = 0$$

where $\Delta\lambda$, $\Delta\mu$, ΔL, &c., are the corrections to be applied to the assumed values of λ, μ, L, &c, and where $\Delta\varpi$ is the observed minus the computed parallax.

Similarly if

$$G = \frac{\varpi^2(H + \Pi\rho_0)}{uH},$$

then

$$\mathfrak{A} \,.\, \Delta\lambda + \mathfrak{B} \,.\, \Delta\mu + A \,.\, \Delta L + B \,.\, \Delta M + C \,.\, \Delta V + D \,.\, \Delta\Lambda + E \,.\, \Delta\Pi + G \,.\, \Delta\frac{d\rho}{dt} = 0$$

where $\Delta\frac{d\rho}{dt}$ is the observed minus the computed radial velocity.

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Note I.

The proper motions employed in this article were those found by Dr. Auwers, and were taken from the Greenwich Ten-Year Catalogue of Stars for 1880, excepting:—

No.	Star.	Authority.
2	α Argûs	*Nautical Alm.*, 1896.
4	α Centauri	Elkin, *Mem.*, vol. xlviii. p. 14.
17	γ Orionis	Main, ,, ,, xix. p. 142.
60	δ Herculis	,, ,, ,, xxviii. p. 134.
64	10 Ursæ Maj.	B. A. C., 3059.
67	e Eridani	Elkin, *Mem.*, vol. xlviii. p. 175.
70	δ Equulei	Stone, ,, ,, xxxiii. p. 73.
74	ζ Toucani	Elkin, ,, ,, xlvii. p. 168.
77	ε Indi	,, ,, ,, xlviii. p. 119.
82	1830 Groomb.	Argelander, Gr. Ten-Year Cat., 1880.
83	1618 Groomb.	,, ,, ,,
84	Σ. 1516	,, ,, ,,
85	21, 185 Lalande	,, ,, ,,
86	9352 Lacaille	Gould, *Mem.*, vol. xlviii. p. 140.
87	Σ. 2398	Lamp., *Astr. Nach.*, 2676 and 2807.
88	34 Groomb.	Argelander, Gr. Ten-Year Cat., 1880.
89	21,258 Lalande	,, ,, ,,
90	17,415 Oeltz. Arg.	,, ,, ,,
91	11,677 Oeltz. Arg.	Geelmuyden, *Astr. Nach.*, 2287.

Note II.

The following parallaxes are employed in the above article, the means of different results being taken when possible. For many of them I am indebted to a Catalogue of Stellar Parallax, by the late Dr. Pritchard (*Ast. Obs. made at the Univ. Obs. Oxford*, No. IV.):—

No.	Star.	Parallax.	Authority.
1	Sirius	0″·23	Henderson, *Mem.* vol. xi. p. 239.
		0·27	Abbe, *M.N.* vol. xxviii. p. 6.
		0·193	Gyldén.
		0·381	Gill and Elkin, *Mem.* vol. xlviii. p. 187.
		0·268	Mean.
2	Canopus	0·028	Elkin, *Mem.* vol. xlviii. p. 183.
3	Arcturus	0·127	Peters.
		0·016	Elkin, *Report Yale Coll. Obs.*
		0·072	Mean.
4	α Centauri	0·913	Henderson and Maclear, *Mem.* vol. xii. p. 370.

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No.	Star.	Parallax.	Authority.
		0″·919	Maclear, *Mem.* vol. xx. p. 98.
		0·916	Mean from zenith distances.
		0·880	Moesta at Santiago,
		0·521	„ *Observatory,* vol. iii. p. 662.
		0·512	Elkin, *M.N.* vol. xli. p. 223.
		0·752	Gill and Elkin, *Mem.* vol. xlviii. p. 186.
		0·716	Mean.
5	Capella	0·046	Peters.
		0·095	Elkin, *Yale Coll. Obs.*
		0·070	Mean.
6	Rigel	−0·004	Gill } *Cat. Stellar Par.*
		0·055	Schur }
		0·026	Mean.
7	Vega	0·06 ?	Airy, *Mem.* vol. x. p. 265.
		0·155	W. Struve, Peters, and O. Struve, *M. N.* vol. xiii. p. 74.
		0·103	Peters, *Cosmos,* vol. iii. p. 261.
		0·188	Brünnow, *Dunsink Obs.* vol. ii.
		0·134	A. Hall, *Nature,* vol. xxxv. p. 258.
		0·092	Elkin, *Yale Coll. Obs.*
		0·122	Mean.
8	Procyon	0·123	Auwers.
		0·341	Elkin, *Yale Coll. Obs.*
		0·232	Mean.
9	α Orionis	0·022	Elkin, *Yale Coll. Obs.*
10	Altair	0·181	Struve, *Mem.* vol. xii. p. 29.
		0·214	Elkin, *Yale Coll. Obs.*
		0·198	Mean.
11	Aldebaran	0·102	A. Hall, *Nature,* vol. xxxvi. p. 138.
		0·101	Elkin, *Yale Coll. Obs.*
		0·102	Mean.
13	Pollux	0·057	Elkin, *Yale Coll. Obs.*
14	Regulus	0·089	„ „ „
15	α Cygni	0·012	„ „ „
20	ε Ursæ Maj.	0·081	Pritchard.
21	α Persei	0·087	„
22	β Tauri	0·063	„
23	α Ursæ Maj.	0·046	„

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No.	Star.	Parallax.	Authority.
25	β Aurigæ	0″·062	Pritchard.
26	α Androm.	0·058	,,
27	Polaris	0·067	Peters.
		0·025	Schweizer.
		0·015	L. de Ball, *Observatory*, vol. viii. p. 314.
		0·079	Pritchard.
		0·046	Mean.
29	α Arietis	0·080	Pritchard.
31	γ Cassiop.	0·012	,,
32	β Androm.	0·074	,,
35	β Leonis	0·029	,,
37	β Ursæ Min.	0·022	,,
40	γ Cygni	0·102	,,
41	β Cassiop.	0·164	,,
42	α Cassiop.	0·036	,,
43	β Persei	0·058	,,
		0·036	Chase, *Observatory*, vol. xvii. p. 253.
		0·047	Mean.
44	β Ursæ Maj.	0·080	Pritchard.
45	γ Ursæ Maj.	0·099	,,
46	γ Draconis	0·092	Molyneux, *Nature*, vol. xviii. p. 669.
		0·050	Pritchard.
		0·071	Mean.
47	ε Pegasi	0·080	Pritchard.
48	α Pegasi	0·082	,,
50	α Cephei	0·062	,,
54	ε Cygni	0·128	,,
57	ι Ursæ Maj.	0·133	Peters.
58	θ Ursæ Maj.	0·052	Kapteyn, *Cat. Stellar Par.*
59	α Herculis	0·050	Jacob, *Mem.* vol. xxviii. p. 45.
60	δ Herculis	0·084	Leavenworth, *Cat. Stellar Par.*
61	π Herculis	0·11	Belopolsky, ,, ,, ,,
62	η Cassiop.	0·234	Struve, ,, ,, ,,
		0·257	Schweizer, *M.N.* vol. lv. p. 341.
		0·443	Davis, ,, ,, ,,
		0·311	Mean.
63	η Herculis	0·40	Belopolsky, *Cat. Stellar Par.*
64	10 Ursæ Maj.	0·20	,, ,, ,, ,,

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No.	Star.	Parallax.	Authority.
65	70 (*p*) Ophi.	0″·150	Krüger.
		0·286	Schur, *Observatory*, vol. xvii. p. 252.
		0·218	Mean.
66	θ Cassiop.	0·232	Jacoby, *Annals New York Academy of Sciences*, vol. viii.
67	e Eridani	0·136	Elkin, *Mem.* vol. xlviii. p. 178.
68	o^2 Eridani	0·166	Gill, ,, ,, ,, p. 160.
		0·223	A. Hall, *Observatory*, vol. viii. p. 345.
		0·194	Mean.
69	δ Ursæ Min.	0·034	L. de Ball, *Observatory*, vol. viii. p. 314.
70	δ Equulei	0·035	Leavenworth, *Cat. Stellar Par.*
71	σ Draconis	0·246	Brünnow, *Dunsink Obs.* vol. i.
72	ν^2 Draconis	0·28	Belopolsky, *Cat. Stellar Par.*
73	ν^1 Draconis	0·32	,, ,, ,, ,,
74	ζ Toucani	0·057	Elkin, *Mem.* vol. xlviii. p. 171.
75	61 Cygni	0·374	Bessel corrected by Peters.
		0·349	Peters, *Cosmos*, vol. iii. p. 260.
		0·505	O. Struve, *Cat. Stellar Par.*
		0·553	Auwers, *M.N.* vol. xxiii. p. 74.
		0·414	Schweizer, *Cat. Stellar Par.*
		0·416	Socoloff, ,, ,, ,,
		0·466	Ball, *M.N.* vol. xl. p. 250 and vol. xli. p. 164.
		0·525	Belopolsky, *Cat. Stellar Par.*
		0·270	A. Hall, *Nature*, vol. xxxv. p. 258.
		0·434	Pritchard.
		0·431	Mean.
76	μ Cassiop.	0·342	O. Struve.
		0·084	Schweizer, *Cat. Stellar Par.*
		0·036	Pritchard.
		0·275	Jacoby, *Annals New York Academy Sc.* vol. viii.
		0·184	Mean.
77	ε Indi	0·215	Gill and Elkin, *Mem.* vol. xlviii. p. 187.
78	Cephei 51 (Hev)	0·027	L. de Ball, *Observatory*, vol. viii. p. 314.
79	3077 Bradley	0·205	Backlund, *M.N.* vol. xliii. p. 223.
		0·283	Gyldén, ,, ,, ,,
		0·070	Brünnow.
		0·185	Mean.

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No.	Star.	Parallax.	Authority.
80	85 Pegasi	0″·058	Brünnow, *Dunsink Obs.* vol. ii.
81	20 Leonis Min.	0·062	Kapteyn, *Cat. Stellar Par.*
82	1830 Groomb.	0·226	Peters, *M.N.* vol. xiv. p. 138.
		0·182	Schluter, *M.N.* vol. xiii. p. 132.
		0·140	Wichmann, *M.N.* vol. xiv. p. 138.
		0·141	Döllen, *Cat. Stellar Par.*
		0·034	O. Struve, *M.N.* vol. x. p. 124.
		0·088	Brünnow, *Dunsink Obs.*
		0·126	Kapteyn, *Cat. Stellar Par.*
		0·134	Mean.
83	1618 Groomb.	0·324	Ball, *M.N.* vol. xlv. p. 254.
		0·176	Kapteyn, *Cat. Stellar Par.*
		0·250	Mean.
84	Σ 1516	0·199	Winnecke and Berberich, *Nature*, vol. xxxv. p. 546.
		0·104	L. de Ball, *Nature*, vol. xxxv. p. 546.
		0·152	Mean.
85	21, 185 Lalande	0·501	Winnecke, *Cat. Stellar Par.*
		0·428	Kapteyn, ,, ,, ,,
		0·464	Mean.
86	9352 Lacaille	0·285	Gill, *Mem.* vol. xlviii. p. 154.
87	Σ 2398	0·353	Lamp, *Astr. Nach.* 2676 and 2807.
88	34 Groomb.	0·307	Auwers, *M.N.* vol. xxviii. p. 97.
89	21, 258 Lalande	0·262	,, ,, ,, xxiii. p. 74.
		0·260	Krüger, ,, ,, ,, p. 172.
		0·168	Kapteyn, *Cat. Stellar Par.*
		0·230	Mean.
90	17, 415 Oeltz. A.	0·247	Krüger, *M.N.* vol. xxiii. p. 172.
		0·151	Schweizer, *Cat. Stellar Par.*
		0·199	Mean.
91	11, 677 Oeltz. A.	0·256	Geelmuyden, *Astr. Nach.* 2287.
		0·100	Franz, *Königsberg Obs.* vol. xxxviii.
		0·178	Mean.

Note III.

In the following table the radial velocities ascribed to Dr. Huggins were taken from the *Monthly Notices*, vol. xxxii., p. 361; those ascribed to Greenwich were taken from the Revision of 1887; and those ascribed to Potsdam from the *Monthly Notices*, vol. lii. p. 541.

In the case of any star the mean of the results found by Dr. Huggins and at Greenwich was taken when possible, and considered to be the result of visual observation, and then the mean of this result and that of photography as found at Potsdam was adopted. In the last column $\frac{d\rho}{dt}$ is expressed as usual in mean radii of the earth's orbit per annum, and it is found by dividing miles per second by 2·9.

No.	Star.	Huggins. Miles per sec.	Greenwich. Miles per sec.	Potsdam. Miles per sec.	Adopted. Miles per sec.	$\frac{d\rho}{dt}$.
1	Sirius	+20	0	− 9·8	0·0	0?
3	Arcturus	−55	−46·0	− 4·8	−27·4	− 9
5	Capella	+	+26·0	+15·2	+20·6	+ 7
6	Rigel	+15	+19·1	+10·2	+13·6	+ 4
7	Vega	−49	−36·1	− 9·5	−26·0	− 9
8	Procyon	+	+15·5	− 5·7	+ 4·9	+ 2
9	α Orionis	+22	+26·9	+10·7	+17·6	+ 6
10	Altair	...	−24·9	−22·9	−23·9	− 8
11	Aldebaran	+	+32·0	+30·2	+31·1	+11
12	Spica	+	−11·9	− 9·2	−10·6	− 4
13	Pollux	−49	−31·8	+ 0·7	−19·8	− 7?
14	Regulus	+14	+10·1	− 5·7	+ 3·2	+ 1
15	α Cygni	−39	−34·1	− 5·0	−20·8	− 7
16	Castor	+26	+17·8	−18·4	+ 1·8	+ 1?
17	γ Orionis	...	...	+ 5·7	+ 5·7	+ 2
18	ϵ Orionis	...	...	+16·5	+16·5	+ 6
19	ζ Orionis	...	...	+ 9·3	+ 9·3	+ 3
20	ϵ Ursæ Maj.	+19	+11·8	−18·8	− 1·7	− 1?
21	α Persei	...	−27·7	− 6·4	−17·0	− 6
22	β Tauri	...	−14·3	+ 5·0	− 4·6	− 2?
23	α Ursæ Maj.	−48	...	− 7·2	−27·6	− 9
24	η Ursæ Maj.	+	...	−16·3	−16·3	− 6
25	β Aurigæ	...	...	−17·5	−17·5	− 6
26	α Androm.	−	−28·2	+ 2·8	−12·7	− 4?
27	Polaris	...	...	−16·1	−16·1	− 6
28	γ Androm.	...	...	− 8·0	− 8·0	− 3
29	α Arietis	...	−14·2	− 9·2	−11·7	− 4
30	γ Gemin.	...	...	−10·3	−10·3	− 4
31	γ Cassiop.	+	−19·3	− 2·2	−10·8	− 4
32	β Androm.	...	...	+ 7·0	+ 7·0	+ 2
33	δ Orionis	...	...	+ 0·6	+ 0·6	0
34	γ Leonis	−	−26·2	−24·0	−25·1	− 9
35	β Leonis	+	...	− 7·6	− 7·6	− 3

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No.	Star.	Huggins. Miles per sec.	Greenwich. Miles per sec.	Potsdam. Miles per sec.	Adopted. Miles per sec.	Adopted. $\frac{d\rho}{dt}$.
36	ζ Ursæ Maj.	...	+19·3	−19·4	0·0	0?
37	β Ursæ Min.	...	...	+ 8·9	+ 8·9	+ 3
38	α Ophiuchi	...	−10·9	+11·9	+ 0·5	0?
39	α Coronæ	+	+20·7	+19·9	+20·3	+ 7
40	γ Cygni	−	−13·5	− 4·0	− 8·8	− 3
41	β Cassiop.	...	− 1·2	+ 3·2	+ 1·0	0
42	α Cassiop.	...	...	− 9·5	− 9·5	− 3
43	β Persei	...	...	− 1·0	− 1·0	0
44	β Ursæ Maj.	+	+22·9	−18·2	+ 2·4	+ 1?
45	γ Ursæ Maj.	+	+17·5	−16·5	+ 0·5	0?
46	γ Draconis	...	− 7·2	...	− 7·2	− 2
47	ε Pegasi	...	−14·3	+ 5·0	− 4·6	− 2?
48	α Pegasi	−	−25·5	+ 0·8	−12·4	− 4?
49	ε Boötis	−	...	−10·1	−10·1	− 4
50	α Cephei	...	−41·8	...	−41·8	−14
51	β Pegasi	...	...	+ 4·1	+ 4·1	+ 1
52	β Libræ	...	...	− 6·0	− 6·0	− 2
53	α Serpentis	...	...	+14·0	+14·0	+ 5
55	δ Leonis	+	...	− 8·9	− 8·9	− 3
56	β Herculis	...	...	−22·0	−22·0	− 8

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On the curve $y = \left\{\frac{1}{x^2 + \sin^2 \psi}\right\}^{\frac{3}{2}}$, *and its connection with an Astronomical Problem.* By Mrs. W. H. Young (Miss Grace Chisholm).

(*Communicated by Professor G. H. Darwin.*)

Introduction. The Problem and its Origin.

The problem of the determination of the orbit of a comet (or other heavenly body) from three observations leads to the study of the following equation :—

$$m - z = \frac{m}{(1 - 2z\cos\psi + z^2)^{\frac{3}{2}}} = \frac{m}{\lambda^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Here m and ψ are certain constants calculated from the observations ; ψ, being the apparent angular distance between the comet and the Sun, lies between 0° and 180° ; m may have any value, but the sign of m is positive or negative according as the apparent path of the comet is convex or concave toward the Sun.*

The unknown quantity z is the ratio of the geocentric distances of the comet and Sun at the time of the mean observation. Hence we perceive that we wish to determine for z a real positive value. The quantity λ, which is obviously always real, is likewise to be taken positive, for it represents the ratio of the heliocentric distances of the comet and Earth at the time of the mean observation.

If we rationalise the equation by squaring both sides we obtain an equation of the eighth degree in z, which, as such, possesses eight complex solutions, conjugate in pairs. One of these is, however, obviously zero ; hence at least one of the other seven solutions is real. The solution $z = 0$ corresponds to the case where we have been observing, not a heavenly body at all, but a speck in our own atmosphere, or even in the telescope itself ; a mistake which is, theoretically at least, always possible.

We have, then, seven solutions to take into account, of which an odd number $(> 0 \overset{=}{<} 7)$ are real. Possibly no one of these real solutions is positive ; in this case the calculations of m and ψ from the observations, or ultimately the observations themselves,

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* $m = \frac{\tau_{,}\tau_{,,,}}{2} \; \frac{\sin P_{,,}}{\sin p_{,,}} \; \frac{1}{R_{,,}^{3}}$,

$P_{,,}$ and $p_{,,}$ being the perpendiculars from the Sun and comet respectively at the time of the mean observation on the great circle through the extreme positions of the comet ; $R_{,,}$ is the distance at the same time of the Earth and Sun ; $\tau_{,}$ is the interval of time between the first and second observations multiplied by Gauss's constant ; and $\tau_{,,,}$ is the same quantity for the second and third observations (Oppolzer, p. 362).